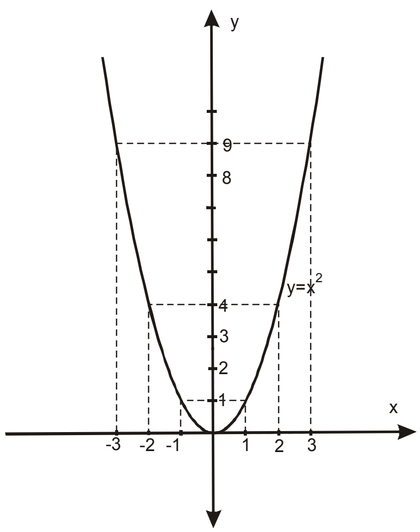
SQUARE FUNCTION $y = ax^2 + bx + c$

It is : $y = ax^2 + bx + c$ where is $x \in R$, $a \neq 0$ and a, b, c are real numbers.

Curve in the plane, which represents a graph function $y = ax^2 + bx + c$ is a **parable.**

First, we learn how to design graph of functions $y = x^2$ .Create table for some value of variable x.

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9



For $x = -3$

$y = (-3)^2 = 9$

For $x = 2$

$y = 2^2 = 4$

For $x = -2$

$y = (-2)^2 = 4$

For $x = 3$

$y = 3^2 = 9$

For $x = -1$

$y = (-1)^2 = 1$

For $x = 0$

$y = 0^2 = 0$

For $x = 1$

$y = 1^2 = 1$

Learn now graph $y = ax^2$

There are 2 situations: $a > 0$ and $a < 0$

For $a > 0$

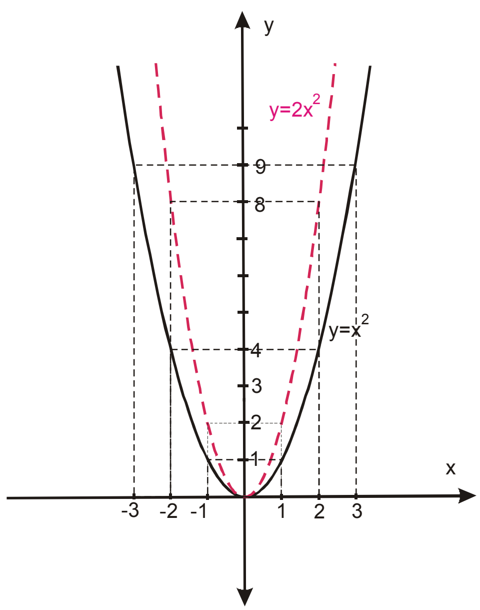
Here is a parable with"opening up". What happens if $a > 1$ and if $0 < a < 1$?

$a > 1$

In relation to the initial graph $y = x^2$, graph $y = ax^2$ ($a > 1$) **is "more thin"**

Example: $y = 2x^2$

x	-3	-2	-1	0	1	2	3
y	18	8	2	0	2	8	18

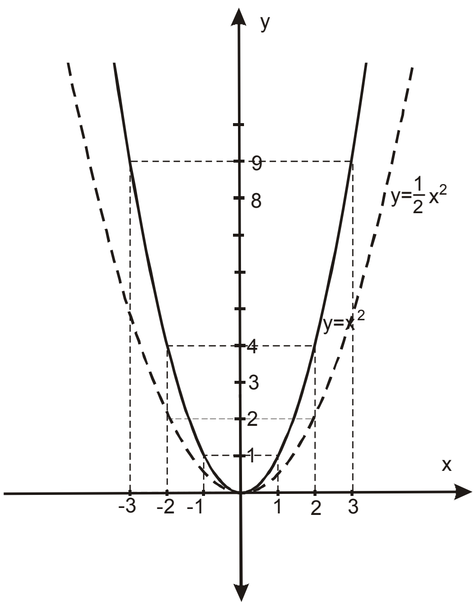


$0 < a < 1$

In relation to the initial graph $y = x^2$, graph $y = ax^2$ $(0 < a < 1)$ is **" more fat"**

Example: $y = \frac{1}{2}x^2$

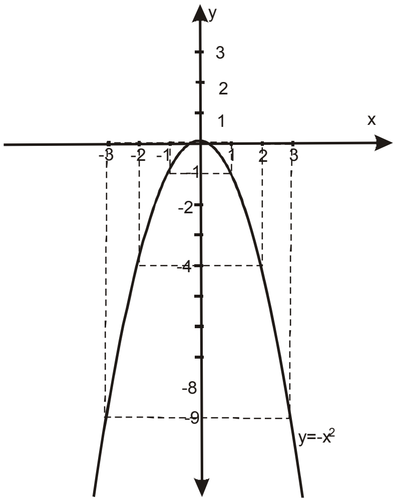
x	-3	-2	-1	0	1	2	3
y	$\frac{9}{2}$	2	$\frac{1}{2}$	0	$\frac{1}{2}$	2	$\frac{9}{2}$



For $a < 0$ here is a parable with"the opening down."

Initial graph is $y = -x^2$.

x	-3	-2	-1	0	1	2	3
y	-9	-8	-2	0	-2	-8	-9

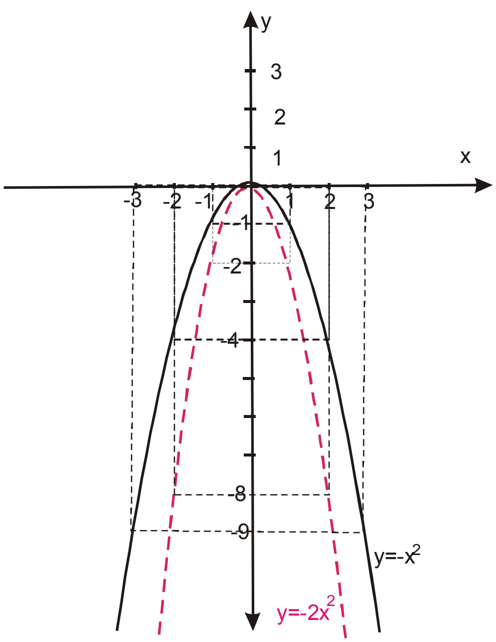


Again, we consider 2 situations for function $y = ax^2$ **: for a < - 1 and - 1 < a < 0**

The following two examples will clarify us their “behavior”:

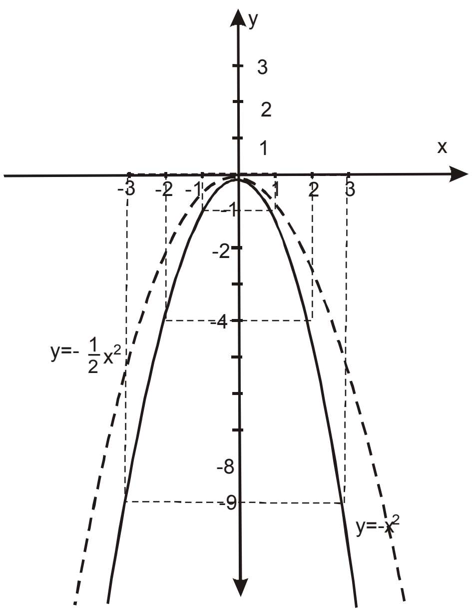
Example 1.: $y = -2x^2$

x	-3	-2	-1	0	1	2	3
y	-18	-8	-2	0	-2	-8	-18



Example 2. : $y = -\frac{1}{2}x^2$

x	-3	-2	-1	0	1	2	3
y	$-\frac{9}{2}$	-2	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-2	$-\frac{9}{2}$



How to draw a graph $y = ax^2 + \beta$?

→ first draft graph of function $y = ax^2$

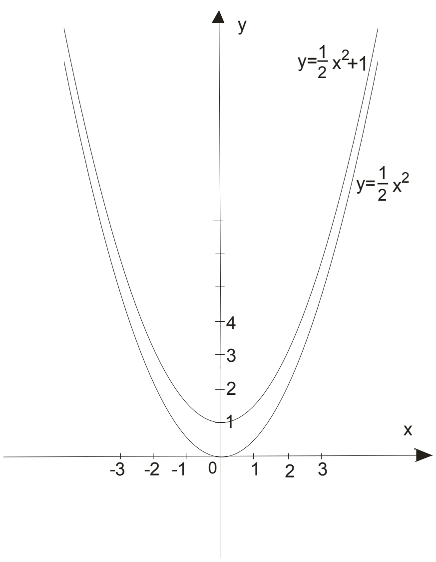
→ that graph then move along y-line for value of β :

1) If β is positive, move it in the positive direction along y- line

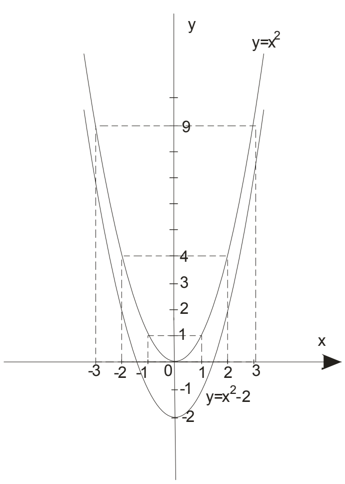
2) If β is negative, move it in the negative direction along y- line

Here are a few examples:

Example 1. $y = \frac{1}{2}x^2 + 1$ (here is β = 1, and we move "up")



Example 2. $y = x^2 - 2$ (here is β = -2 and we move "down")



How to draw a graph $y=(x-\alpha)^2$?

→ first draft graph of function $y=x^2$ (or $y=-x^2$)

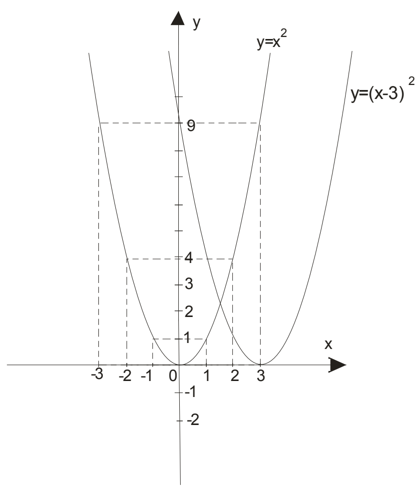
→ that graph then move along x-line :

1) If we have x + α, move it in the negative direction along x- line (α is negative)

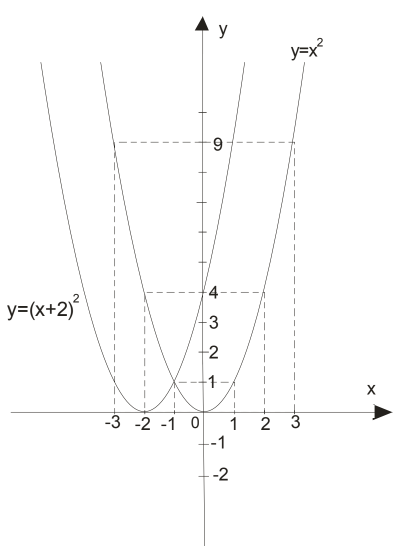
2) If we have x - α, move it in the positive direction along x- line(α is positive)

Here are a few examples:

Example 1. $y=(x-3)^2$ (here is α = 3, and we move in positive direction)



Example 2. $y=(x+2)^2$ (here is α = - 2, and we move in negative direction)



Now we have the knowledge to draft the graph of function $y = ax^2 + bx + c$

First, we have to use so-called "canonical form" : $y = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$

Or, if we mark $\alpha = -\frac{b}{2a}$ and $\beta = \frac{4ac - b^2}{4a}$, we have : $y = a(x-\alpha)^2 + \beta$ and thih form we will use.

Important procedure

i) **Put** $y = ax^2 + bx + c$ **in canonical form** : $y = a(x-\alpha)^2 + \beta$

ii) **Draw graph of function** $y = ax^2$

iii) **Process move along the x-line for α**

iv) **Process move along the y-line for β**

Example: Draw a graph $y = x^2 - 6x + 5$

Solution:

i) **Put** $y = ax^2 + bx + c$ **in canonical form** : $\boxed{y = a(x-\alpha)^2 + \beta}$

$a = 1$

$b = -6$

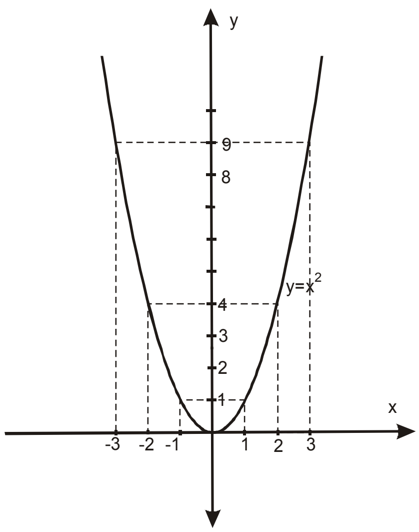
$c = 5$

$$\alpha = -\frac{b}{2a} = -\frac{-6}{2 \cdot 1} = 3$$

$$\beta = \frac{4ac - b^2}{4a} = \frac{4 \cdot 1 \cdot 5 - (-6)^2}{4 \cdot 1} = \frac{20 - 36}{4} = \frac{-16}{4} = -4$$

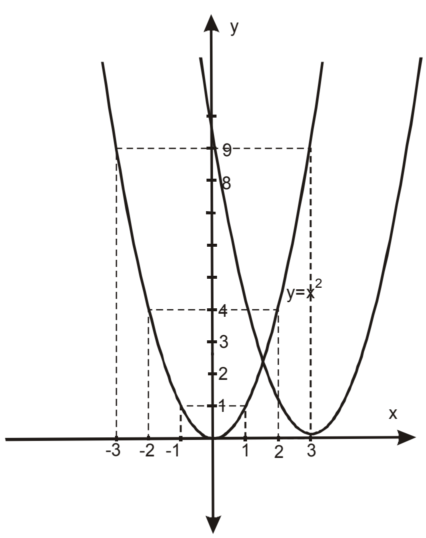
$$y = a(x-\alpha)^2 + \beta$$
$$y = 1(x-3)^2 + (-4)$$
$$y = (x-3)^2 - 4$$

ii) Draw graph of function $y = ax^2$ $\longrightarrow$ $y = x^2$ Because (a=1)



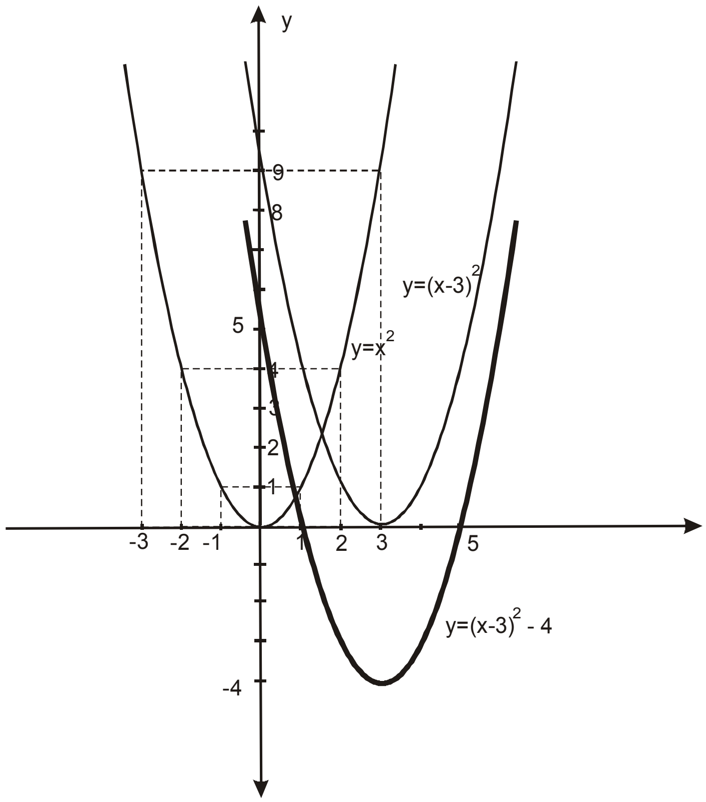
iii) Process move along the x-line for α

$y = x^2 \longrightarrow y = (x-3)^2$



iv) **Process move along the y-line for β**

$y = (x-3)^2 \longrightarrow y = (x-3)^2 - 4$



The whole process is little "complicated" and not even much accurate. Here's how to more quickly and accurately draw a graph $y = ax^2 + bx + c$ without reducing the canonical form and use other "movements":

Procedure:

1) First, determine *a, b, c* and find $D = b^2 - 4ac$

2) $x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$

$D > 0, x_1 \neq x_2$
$D = 0, x_1 = x_2$
$D < 0, No... x_1, x_2$

3) Depending on the number a, we have:

i) **If** $a > 0$ 

ii) **If** $a < 0$ 

4) Parable always cutting the y-line in point (0,c)

5) $T(\alpha,\beta) \quad \alpha = -\dfrac{b}{2a}, \beta = -\dfrac{D}{4a}$

$T(\alpha,\beta)$ **is max if** $a<0$

$T(\alpha,\beta)$ **is min if** $a>0$

Example 1): $y = x^2 - 6x + 5$

Solution:

1)

$a=1 \qquad D = b^2 - 4ac = (-6)^2 - 4\cdot 1\cdot 5 = 36 - 20 = 16$

$b=-6$

$c=5$

2)

$x_{1,2} = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{6 \pm 4}{2}$

$x_1 = 5$

$x_2 = 1$

3)

$a = 1 > 0 \Rightarrow$ 

4)

y-line is cutting in c=5

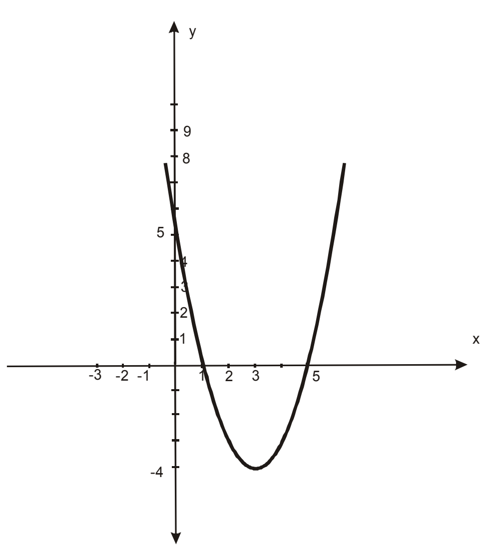
5)

$T(\alpha,\beta)$

$\alpha = -\dfrac{b}{2a} = \dfrac{-6}{2\cdot 1} = 3$

$\beta = -\dfrac{D}{4a} = -\dfrac{16}{4\cdot 1} = -4$

$\underbrace{T(3,-4)}_{\text{min}}$



Example 2) $y = -\frac{1}{2}x^2 + \frac{1}{2}x + 6$

Solution:

1)

$$a = -\frac{1}{2}$$
$$b = \frac{1}{2}$$
$$c = 6$$
$$D = \left(\frac{1}{2}\right)^2 - 4\left(-\frac{1}{2}\right) \cdot 6 = -\frac{1}{4} + 12 = 12\frac{1}{4} = \frac{49}{4}$$

2)

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{\frac{1}{2} \pm \frac{7}{2}}{2\left(-\frac{1}{2}\right)} = \frac{-\frac{1}{2} \pm \frac{7}{2}}{-1}$$
$$x_1 = -3$$
$$x_2 = 4$$

3)

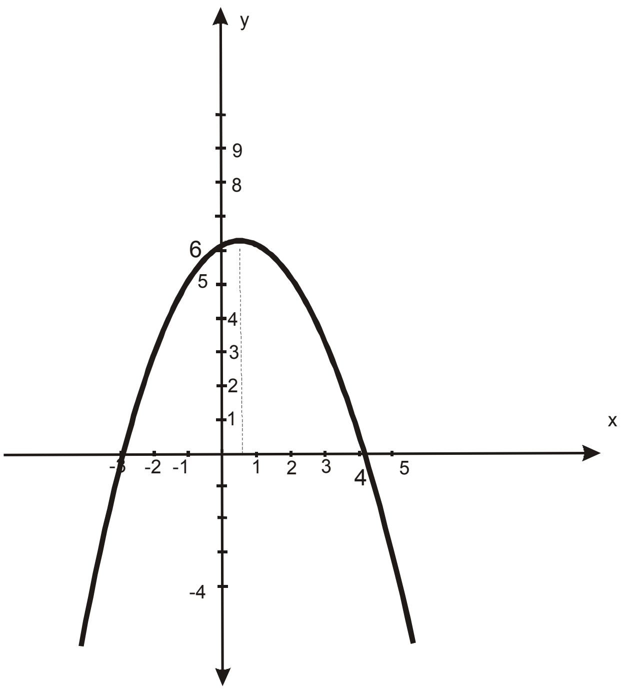
$a = -\frac{1}{2} < 0 \Rightarrow$ 

4)

y-line is cutting in c=6

5)

$$T(\alpha, \beta)$$
$$\alpha = -\frac{b}{2a} = -\frac{\frac{1}{2}}{2\left(-\frac{1}{2}\right)} = \frac{1}{2}$$
$$\beta = -\frac{D}{4a} = -\frac{\frac{49}{4}}{4\left(-\frac{1}{2}\right)} = +\frac{49}{8} = 6\frac{1}{8}$$
$$T\left(\frac{1}{2}, 6\frac{1}{8}\right)$$



Example 3) $y = x^2 - 4|x| + 3$

Solution:

Because $|x| = \begin{cases} x, x \geq 0 \\ -x, x < 0 \end{cases}$ here we have 2 graphics, one for $x \geq 0$, and one for $x < 0$

$$y = x^2 - 4|x| + 3$$

<u>for $x \geq 0$</u>

$$\underline{y = x^2 - 4x + 3}$$

1)

$$a = 1, b = -4, c = 3$$
$$D = b^2 - 4ac = 16 - 12 = 4$$

2)

$$x_{1,2} = \frac{4 \pm 2}{2}$$
$$x_1 = 3$$
$$x_2 = 1$$

3)

$a = 1 > 0 \Rightarrow$ 

4)

y-line is cutting in 3

5)

$T(\alpha,\beta)$

$\alpha = -\frac{b}{2a} = \frac{-4}{2\cdot 1} = 2$

$\beta = -\frac{D}{4a} = -\frac{4}{4\cdot 1} = -1$

$T(2,-1)$

<u>For $x < 0$</u>

<u>$y = x^2 + 4x + 3$</u>

1)

$a = 1, b = 4, c = 3$

$D = b^2 - 4ac = 16 - 12 = 4$

2)

$x_{1,2} = \frac{-4 \pm 2}{2}$

$x_1 = -1$

$x_2 = -3$

3)

$a = 1 > \Rightarrow$ 

4)

y-line is cutting in 3

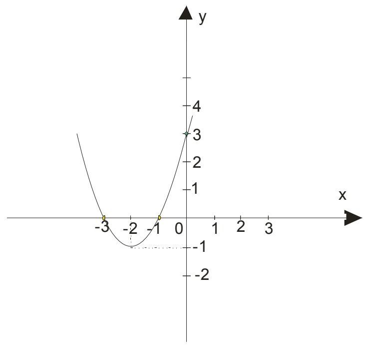
5)

$T(\alpha,\beta)$

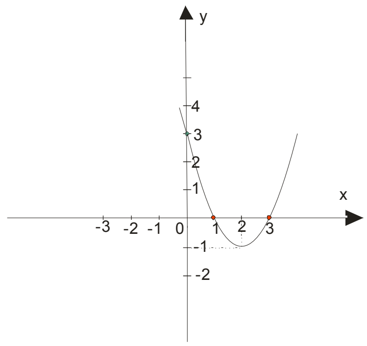
$\alpha = -\frac{b}{2a} = \frac{-4}{2\cdot 1} = -2$

$\beta = -\frac{D}{4a} = -\frac{4}{4\cdot 1} = -1$

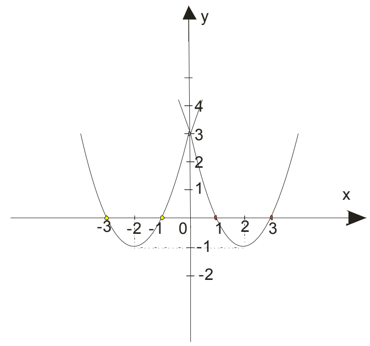
$T(-2,-1)$



$y = x^2 + 4x + 3$



$y = x^2 - 4x + 3$



$y = x^2 - 4|x| + 3$

First graph $y = x^2 - 4x + 3$ we need only for $x \geq 0$ and
second graph $y = x^2 + 4x + 3$ for $x < 0$

final solution (FOR $y = x^2 - 4|x| + 3$) is: